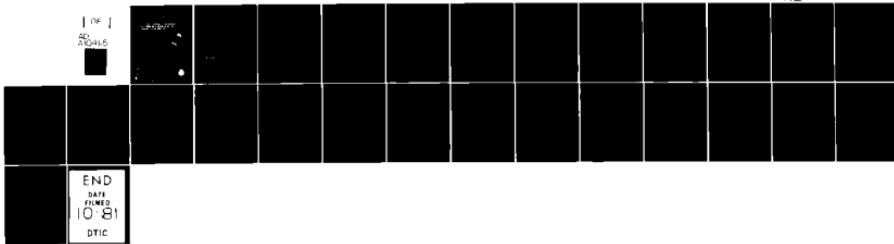


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# A FAST WALSH TRANSFORM ELECTROCARDIOGRAM DATA COMPRESSION ALGORITHM SUITABLE FOR MICROPROCESSOR IMPLEMENTATION

Walter S. Kuklinski, Ph.D.  
Al M. Zied, Ph.D.

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Brooks Air Force Base, Texas 78235



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This technical report has been reviewed and is approved for publication.

*AL M. ZIED*  
AL M. ZIED, Ph.D.  
Project Scientist

*James R. Hickman*  
JAMES R. HICKMAN, JR., Colonel, USAF, MC  
Supervisor

*R.L. DeHart*  
ROY L. DEHART  
Colonel, USAF, MC  
Commander

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## A FAST WALSH TRANSFORM ELECTROCARDIOGRAM DATA COMPRESSION ALGORITHM SUITABLE FOR MICROPROCESSOR IMPLEMENTATION

### INTRODUCTION

The advantages of digital transmission, storage, and processing of electrocardiogram (ECG) data have been documented in many studies [1-3]. One factor limiting a more widespread application of these techniques is the relatively large number of bits required to adequately represent an ECG. Using American Heart Association standards [4] of 500 samples/second with 9 bits per sample, a three-lead vectorcardiogram represents a data rate of 13,500 bits per second (BAUD) and requires approximately 18,000 8-bit words to store a 10-second data record.

One method of reducing these data transmission and storage requirements is by utilizing a data compression algorithm. The operation of a data compression algorithm is illustrated in Figure 1. The original signal  $Y(n)$ , an  $N$  element long sequence of  $M$  bit binary numbers, is operated on by  $X_0(n)$  producing  $Y_C(n)$ , a compressed representation of  $Y(n)$ .  $Y_C(n)$  has  $N_0$  elements with  $M_0$  bits/element where  $N_0M_0 < NM$ . The original signal is reconstructed from  $Y_C(n)$  by the process  $X_1(n)$ . This reconstructed output sequence can be represented as the sum of the input sequence  $Y(n)$  and an error sequence  $E(n)$ .

$$\bar{Y}(n) = Y(n) + E(n) \quad (1)$$

Typically the magnitude of this error sequence is proportional to the data compression ratio  $NM/N_0M_0$ .

In developing the data compression algorithm presented in this report, an additional constraint was considered--namely, eventual implementation of the algorithm in real or pseudo-real time with a microprocessor. A survey of previous work in ECG data compression suggested two possible techniques: direct data compression [5-9] and transformation compression [9-12]. In direct data compression techniques the compression algorithm,  $X_0(n)$ , operates on the original data sequence,  $Y(n)$ , such that the compressed representation of the input sequence,  $Y_C(n)$ , is made up of the actual elements of the input sequence,  $Y(n)$ , or these values within a tolerance. Transformation data compression techniques are those which apply a linear or nonlinear transformation,  $X(n)$ , to the input data sequence,  $Y(n)$ , to produce the compressed representation,  $Y_C(n)$ ; the reconstruction  $Y(n)$  is obtained by applying the inverse transformation  $X^{-1}(n)$  to  $Y_C(n)$ . Examples of transforms used by previous investigators are Fourier transform [9,10], Haar transform [10], and the Karhunen Loeve transform [10-12].

Evaluation of the comparative performance of these two data compression techniques revealed the transformation techniques to be superior in terms of

the quality of the reconstructed signal and the direct data compression techniques to be more suitable for microprocessor implementation. Utilizing these facts the data compression technique studied here was a Walsh transform technique that combined the superior reconstruction characteristics of the transformation data compression techniques with the computational advantages of the direct data compression methods.

The performance of a Walsh transform data compression algorithm, implemented in Fortran on a PDP-11/70 computer, was evaluated using ECG data sampled at 500 Hz using 9 bits/sample. From the limited number of ECG's tested an acceptable reconstructed signal, using the diagnostic content of the signal as an objective criterion, could be obtained at data compression ratios of approximately 4:1. The mean square error between the original and reconstructed signals at this compression ratio was approximately 1%.

#### OBJECTIVES

The main objective of this study was to investigate the operating characteristics of a Fast Walsh transform (FWT) electrocardiogram (ECG) data compression algorithm. Although certain aspects of this data compression algorithm's behavior can only be determined from an actual microprocessor implementation, its basic operational characteristics can be determined from the Fortran minicomputer-based implementation of the algorithm developed in this study. The specific objectives were:

- (1) To determine the relationship between the faithfulness of a reconstructed ECG signal and the number of Walsh functions used in the reconstruction process.
- (2) To determine the relationship between the faithfulness of a reconstructed ECG signal and the number of bits used to represent the magnitude of the Walsh coefficients.
- (3) To determine if filtering the reconstructed ECG signal can improve its diagnostic utility.

#### FAST WALSH TRANSFORM ECG DATA COMPRESSION ALGORITHM

The method of data compression employed in this study was based on the use of an orthogonal signal basis set, namely Walsh functions. To aid in understanding the operation of this data compression technique, a brief review of orthogonal functions, in general, and Walsh functions, in particular, along with a description of the method used to compute a sequency-ordered FWT, will be presented.

if Two functions,  $\phi_1(t)$  and  $\phi_2(t)$ , are orthogonal over the interval  $[t_1, t_2]$

$$\int_{t_1}^{t_2} \phi_1(t)\phi_2(t)dt = 0 \quad (2)$$

Because of this property it is generally possible to represent a function,  $f(t)$ , over a certain interval by a linear combination of mutually orthogonal functions. If the set of functions,  $\phi_1$ , is complete over the interval  $[t_1, t_2]$ , then  $f(t)$  can be expressed as

$$f(t) = \sum_{n=1}^N a_n \phi_n(t) \quad (3)$$

where  $\phi_n$  is the  $n$ th member of a set of mutually orthogonal basis functions. To represent an arbitrary function,  $f(t)$ , an infinite number of basis functions,  $\phi_n(t)$ , may be necessary. A data transmission system utilizing this representation of a signal is shown in Figure 2. Given that the basis functions are known at both the receiver and transmitter, the transmitter computes from the signal the  $N a_n$  terms. These magnitudes are transmitted and used by the receiver to reconstruct the original signal,  $f(t)$ .

In cases where the signal  $f(t)$  exists only at discrete time intervals, as occurs with the sampled ECG signals we are dealing with here, the number of terms in the orthogonal representation of the signal is finite. A compression of the data can therefore be effected in either of two ways: first by transmitting less than the entire  $N a_n$  terms, or by using fewer bits to represent the  $a_n$  terms than were used to represent the original signal. In both cases an error signal will be introduced to the reconstructed signal. By varying both the number of terms and the number of bits used to represent the magnitude of these terms, a limit was obtained as to the extent the data can be compressed and still be of diagnostic use when reconstructed.

In addition to having the properties other orthogonal functions possess, Walsh functions [13-17] have another property; namely, only addition and subtraction of the original signal sample values from each other, in a particular sequence, are necessary to compute a Walsh transform representation of the signal. By employing an FWT algorithm [15-17] that requires  $n \log n$  operations to compute a transform, as compared to  $n^2$  operations using a direct method, the computations necessary to generate a Walsh transform in real time can be performed on most 8-bit microprocessors without requiring a hardware floating point capability.

The FWT algorithm used in this study [17] is as follows. The Walsh-Fourier transform  $F(m)$  of an  $N$  point sampled signal  $F(n)$  is defined as

$$F(m) = \sum_{n=1}^N f(n) \text{Wal}(m,n) \quad \text{for } m = 0, 1, 2, \dots, N-1 \quad (4)$$

with the associated inverse transform

$$f(n) = \frac{1}{N} \sum_{m=0}^{N-1} F(m) \text{Wal}(n,m) \quad \text{for } n = 1, 2, \dots, N$$

The discrete Walsh functions are sampled versions of the continuous set. In Figure 3, the first four continuous Walsh functions are shown. In this figure the index  $n$  is used to order these functions by sequency, that is the number of zero crossings in the interval 0,1. In the discrete form of these functions the second index denotes the sample number. For purposes of the FWT development the Walsh functions are assumed to be periodic with period  $N$  where  $N$  is an integer power of 2. These discrete Walsh functions can be defined as follows:

$$\text{Wal}(0,n) = 1 \text{ for } n = 1, 2, 3, \dots, N \quad (5)$$

$$\text{Wal}(1,n) = \begin{cases} 1 & \text{for } n = 1, 2, \dots, N/2 \\ -1 & \text{for } n = N/2 + 1, N/2 + 2, \dots, N \end{cases} \quad (6)$$

In general,

$$\text{Wal}(m,n) = \text{Wal}([m/2], 2n) \cdot \text{Wal}(m-2[m/2], n) \quad (7)$$

where  $[m/2]$  is the integer part of  $m/2$ .

Using these definitions, the FWT algorithm can be summarized as

$$F_0(n,0) = f(n) \quad \text{for } 1 \leq n \leq N \quad (8)$$

$$F_i(K,J) = F_i(2K-1, [J/2]) + (-1)^{[(J+1)/2]} F_i(2K, [J/2]) \quad (9)$$

for

$$i = 0, 1, 2, \dots, p-1; p = \log_2 N$$

$$J = 0, 1, 2, \dots, 2^{i+1}-1$$

$$K = 1, 2, 3, \dots, \frac{N}{2^{i+1}}$$

These functions can be related to the Walsh function presented in equation 4 as

$$F(j) = F_p(i,j) \quad (10)$$

A Fortran implementation of this FWT algorithm was incorporated in the data compression algorithm used in this study.

#### METHODS AND RESULTS

The ECG data used to study the FWT data compression algorithm had been sampled at 500 Hz with 12 bits used to represent each sample. So that the results obtained here could be compared with previous work in ECG data compression, the data was initially requantized to 9 bits/sample. There were 11 channels of data available: the standard lead I and the lead II data, the 6 precordial leads, and the 3 vector leads. Although the algorithm was tested using all 11 channels, only the lead II data was used for the detailed results presented here. Data from three different patients, all lead II, was processed as follows.

The 12-bit sample 500 sample/second data was requantized to 9 bits/sample and segmented into 512 sample, 1.024 second, records. These records were operated on by the FWT algorithm that produced the 512 Walsh coefficients. The data, represented by these 512 Walsh coefficients, was compressed by the two following methods.

The first method of compressing the data was to quantize the Walsh coefficients to between 11 and 3 bits/coefficent. The upper limit of 11 bits was necessary to reconstruct the ECG signal and not introduce any error. The second method of compressing the data was by using only a fraction of the Walsh coefficients to reconstruct the ECG signal. This fraction was varied from 1 to 1/16; that is, ECG signals were reconstructed using, at one extreme, all 512 Walsh coefficients and, at the other extreme, only 512/16 of the coefficients. The fraction of Walsh coefficients retained was always that fraction having the greatest magnitude. For example, at 512/4 the 128 Walsh coefficients having the largest magnitude were retained. The remaining coefficients were set to zero and the ECG signal reconstructed using the inverse FWT algorithm. The normalized mean square error between the original and reconstructed ECG signals was computed using the following relationship

$$MSE = \frac{\sum_{n=1}^{512} (Y(n) - \bar{Y}(n))^2}{\sum_{n=1}^{512} (Y(n))^2}$$

Plots of the reconstructed ECG signals, lead II, for one patient are seen in Figures 4-8. Each of the five sets of five figures show the reconstructed ECG at a constant coefficient reduction ratio with the number of bits used to represent each coefficient varied from 11 to 3 bits from bottom to top. For example, Figure 6 shows the reconstructed lead II ECG's for patient one where the 128 largest magnitude Walsh coefficients were used in the reconstruction and these coefficients were represented as 11 through 3 bit binary numbers. One can note the degradation of the reconstructed signal as both the number of bits/coefficient and percentage of coefficients retained are reduced.

Figure 9 is a set of contour plots of the normalized mean square error for the set of data presented in Figures 4-8. It shows the mean square error versus the number of bits/coefficient and the ratio of coefficients retained/coefficients zeroed in the reconstruction for patient one, which corresponds to the time waveforms in Figures 4-8.

Figures 10 through 14 show the effects of utilizing a digital 9th-order finite-impulse response low-pass filter on the reconstructed data from patient three. The zero locations of this filter along with  $H(z)$ , its transfer function, can be found in Figure 15. Note the removal of the 60-Hz noise signal from the ECG. The large mean square error is caused by a phase shift (delay) introduced by the filter, not a change in the waveshape.

#### DISCUSSION AND RECOMMENDATIONS

The results obtained in this study demonstrate the utility of a data compression technique based on an FWT algorithm. The relationship between mean square error and both the number of Walsh coefficients used in the ECG reconstruction and the number of bits used to represent each coefficient was a monotonically increasing function of both parameters. This relationship, which held for all three sets of ECG data, can clearly be seen in the contour plots in Figure 9. The trade-off between these two parameters in terms of mean square errors seems to be unimportant; that is, it seems that using half the coefficients or half as many bits to represent all the coefficients has the same effect on the resulting reconstructed ECG in a mean square error sense.

The more important point is: what validity does the mean square error have in terms of being a useful measure of the diagnostic content of the reconstructed ECG waveform? From contacts with Air Force and other cardiologists it was apparent that two reconstructions of an ECG signal with the same mean square error resulted in different diagnoses. A more meaningful criterion for evaluating the utility of a reconstructed ECG waveform should be established. This criterion should be based on the measures used by cardiologists in their evaluation of actual ECG data.

The second area that has to be addressed to complete evaluation of this algorithm would be a microprocessor implementation, so that the real-time operating characteristics can be determined. Once the overall performance of this data compression algorithm has been determined, it can be used as a benchmark against which other data compression algorithms can be compared.

The use of a simple low-pass 9th-order finite impulse response digital filter seemed to improve the diagnostic utility of the reconstructed ECG signal. The use of other filters such as Wiener filters, for example, may

improve the utility of these reconstructed ECG's. A study of the noise introduced by the reconstruction process when using a reduced number of coefficients or a reduced number of bits/coefficients should be undertaken and this information used to determine an optimal filter for this problem.

To summarize these recommendations:

1. Determine the maximum data compression that can be obtained with the FWT algorithm and still yield clinically useful information. This will be done using cardiologists' diagnosis of the reconstructed ECG's. Signals over a wide range of heart rates and from patients with a wide range of conditions should be used.
2. Implement the FWT algorithm through a microprocessor. This will allow the real-time operating characteristics of the algorithm to be determined. These performance characteristics can be used as a benchmark against which other data compression algorithms can be compared.
3. Study the noise characteristics of the reconstructed ECG signals and use this information to develop a filter to optimize the diagnostic utility of these reconstructed ECG signals.

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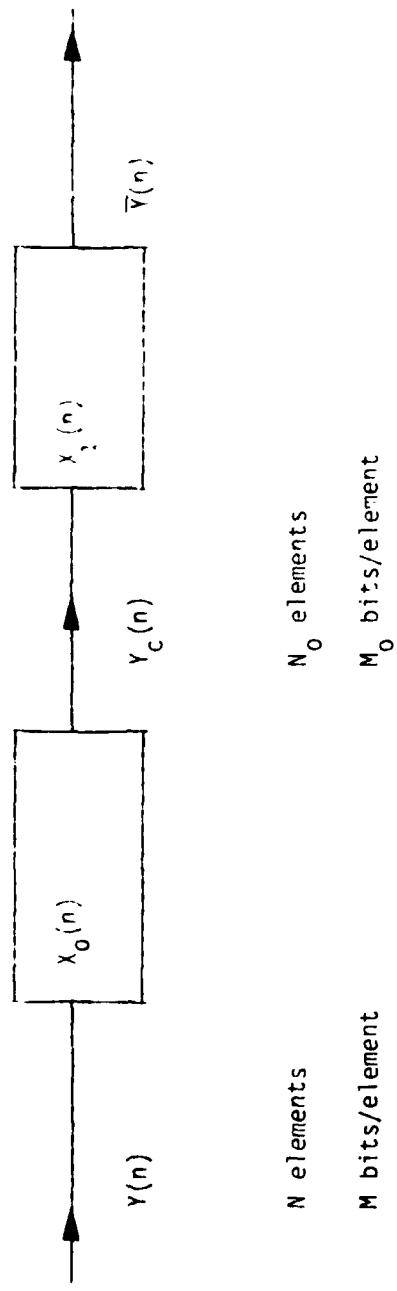


Figure 1. Data compression via data reduction.

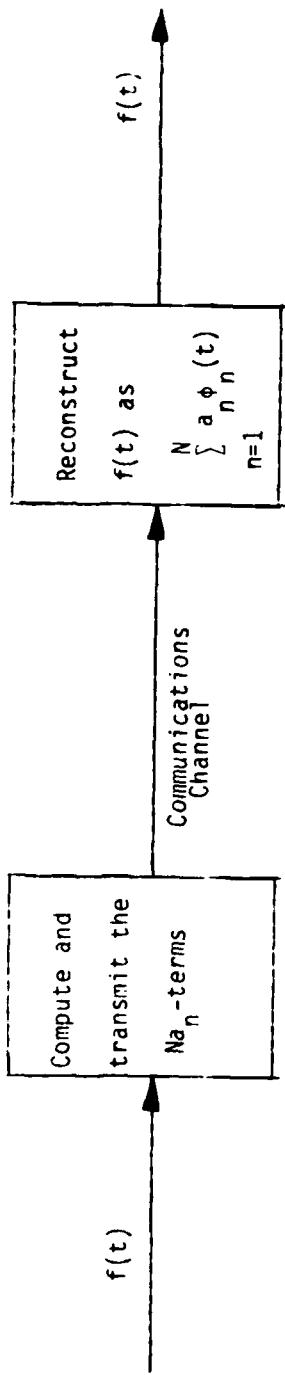


Figure 2. Data transmission system via orthogonal basis function.

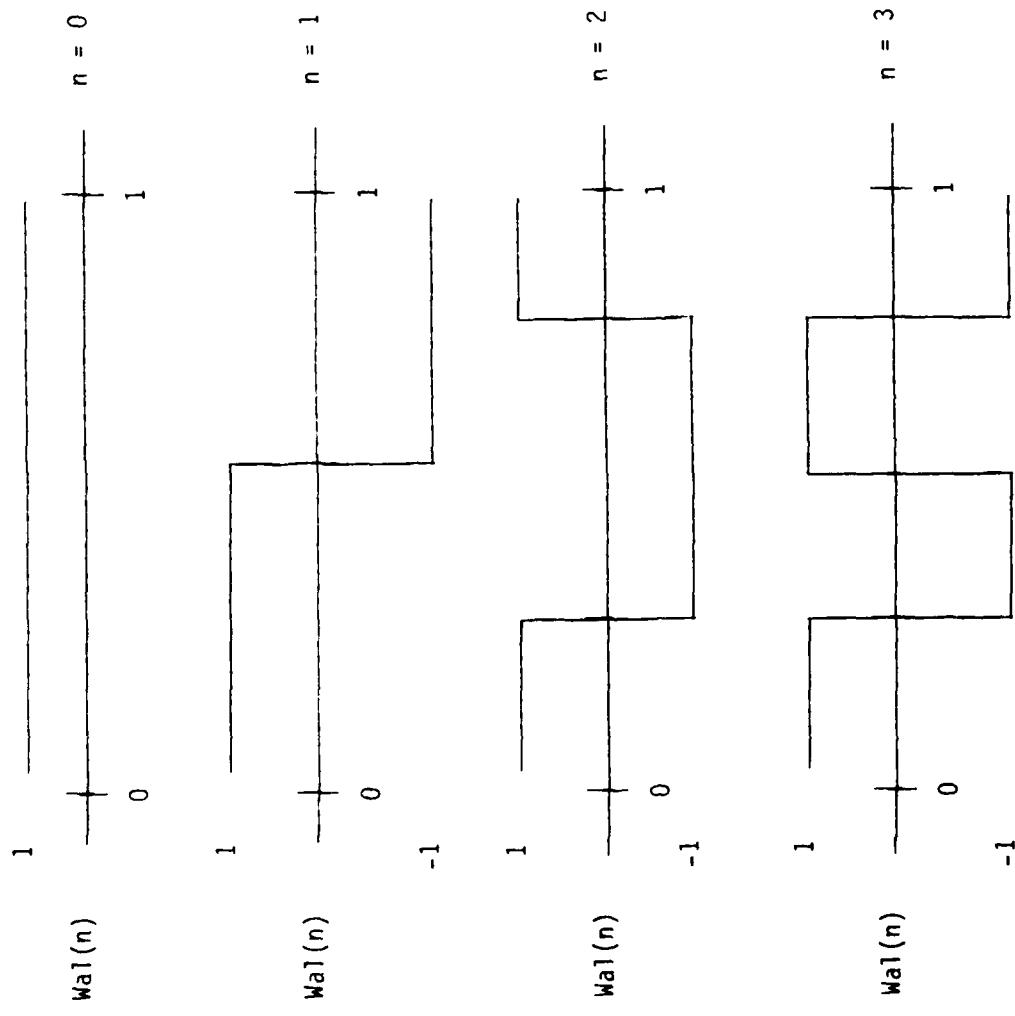


Figure 3. The first four Walsh functions.

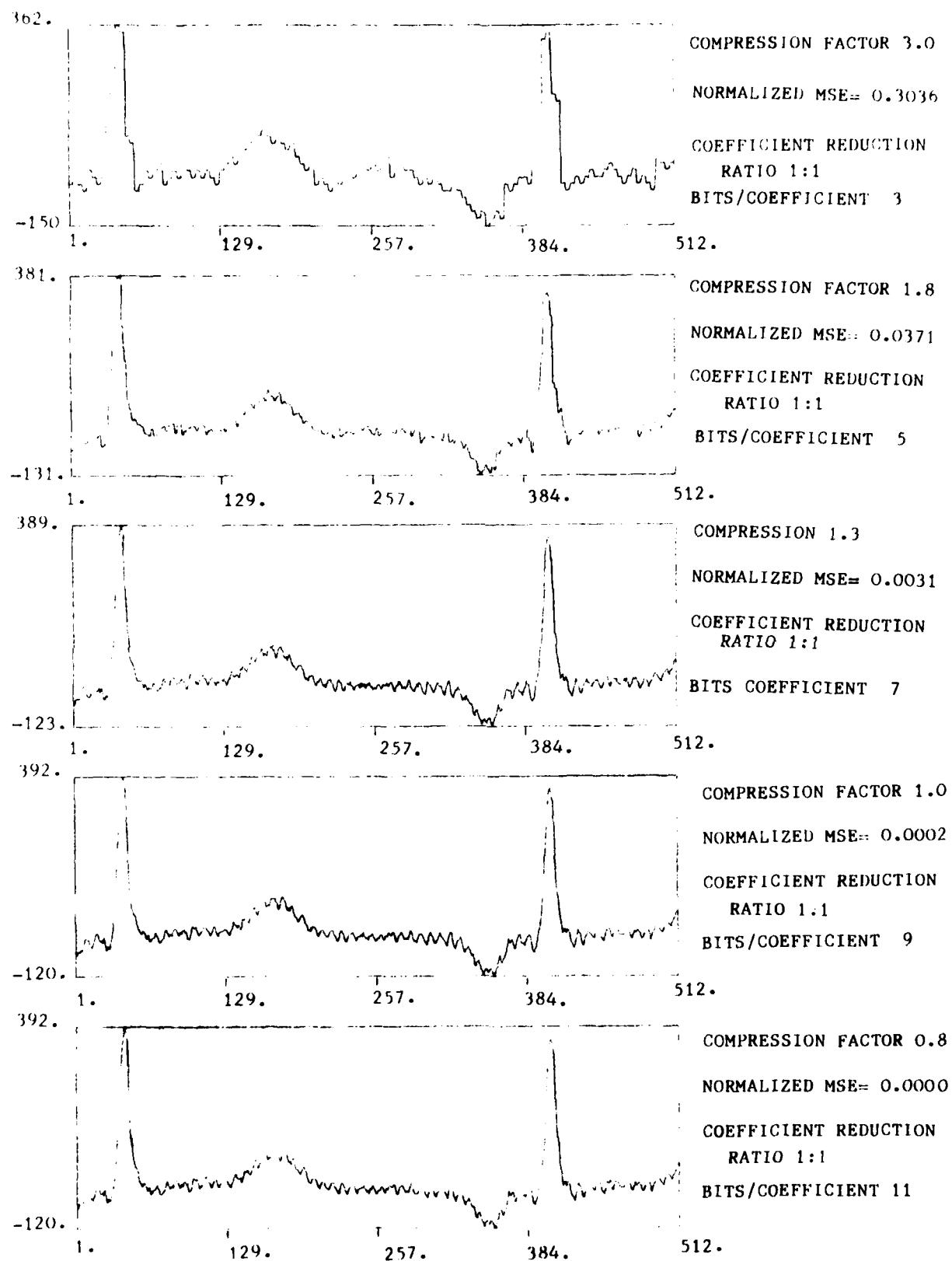


Figure 4. Lead II reconstruction--1:1 coefficient reduction.

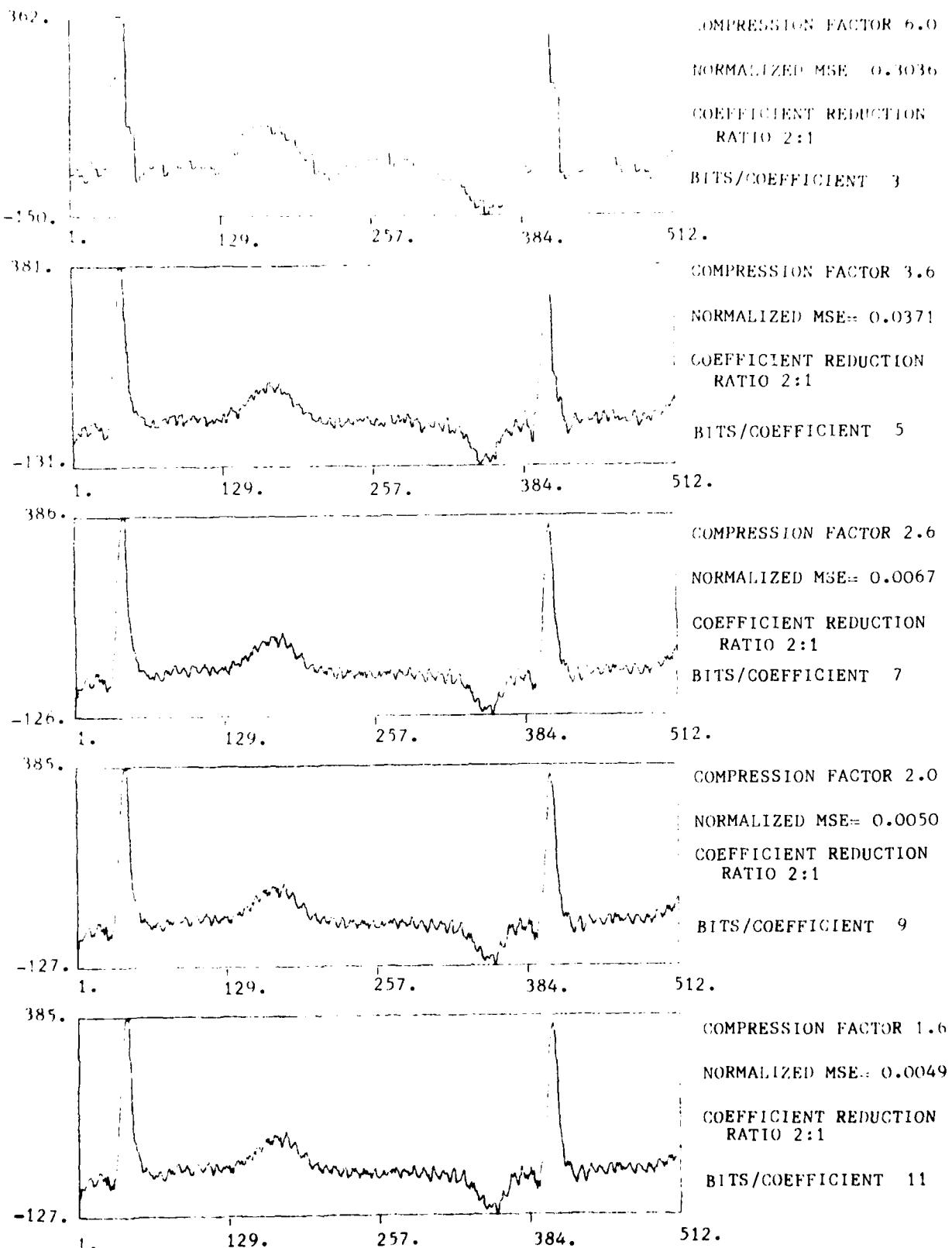


Figure 5. Lead II reconstruction--2:1 coefficient reduction.

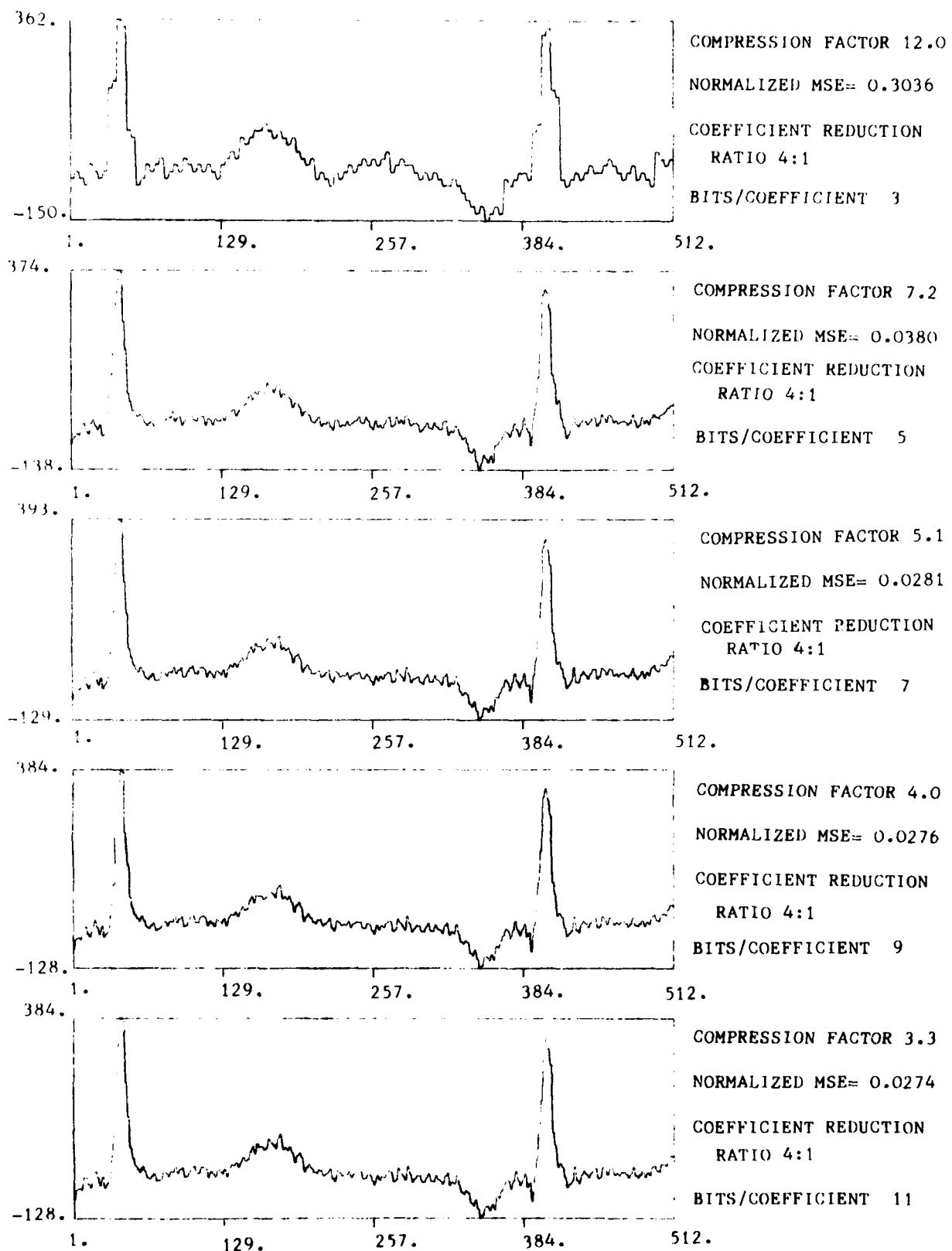


Figure 6. Lead II reconstruction--4:1 coefficient reduction.

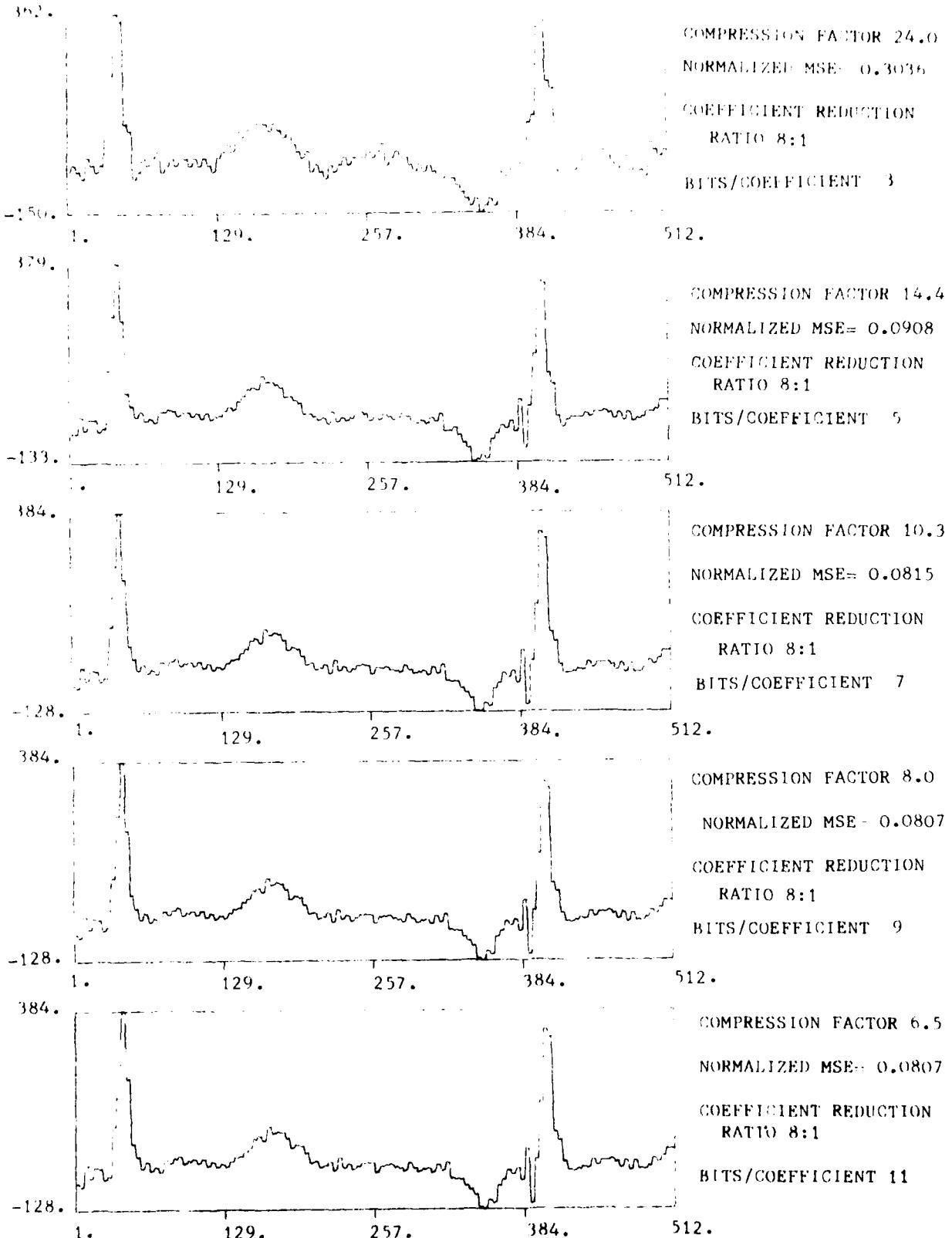


Figure 7. Lead II reconstruction--8:1 coefficient reduction.

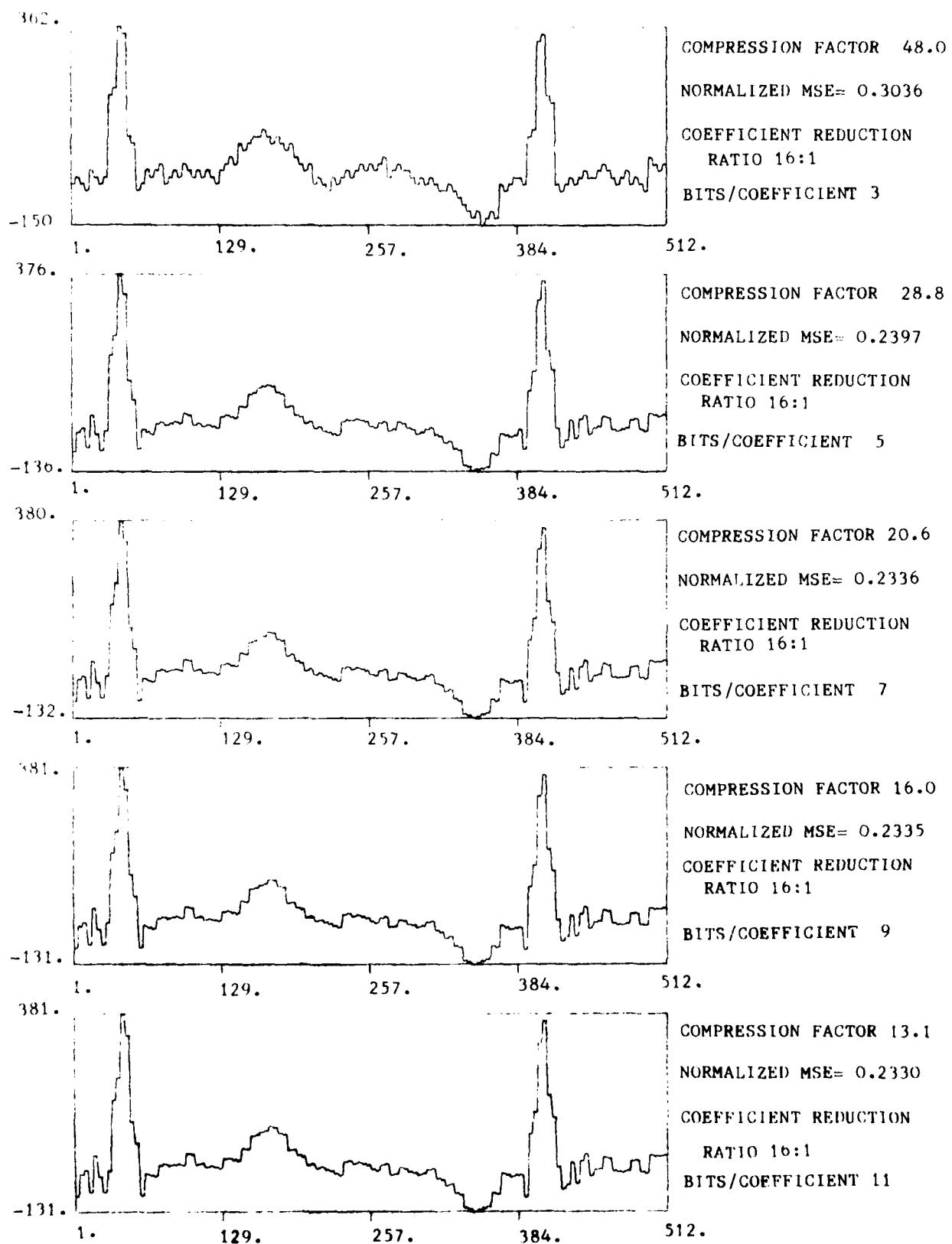


Figure 8. Lead II reconstruction--16:1 coefficient reduction.

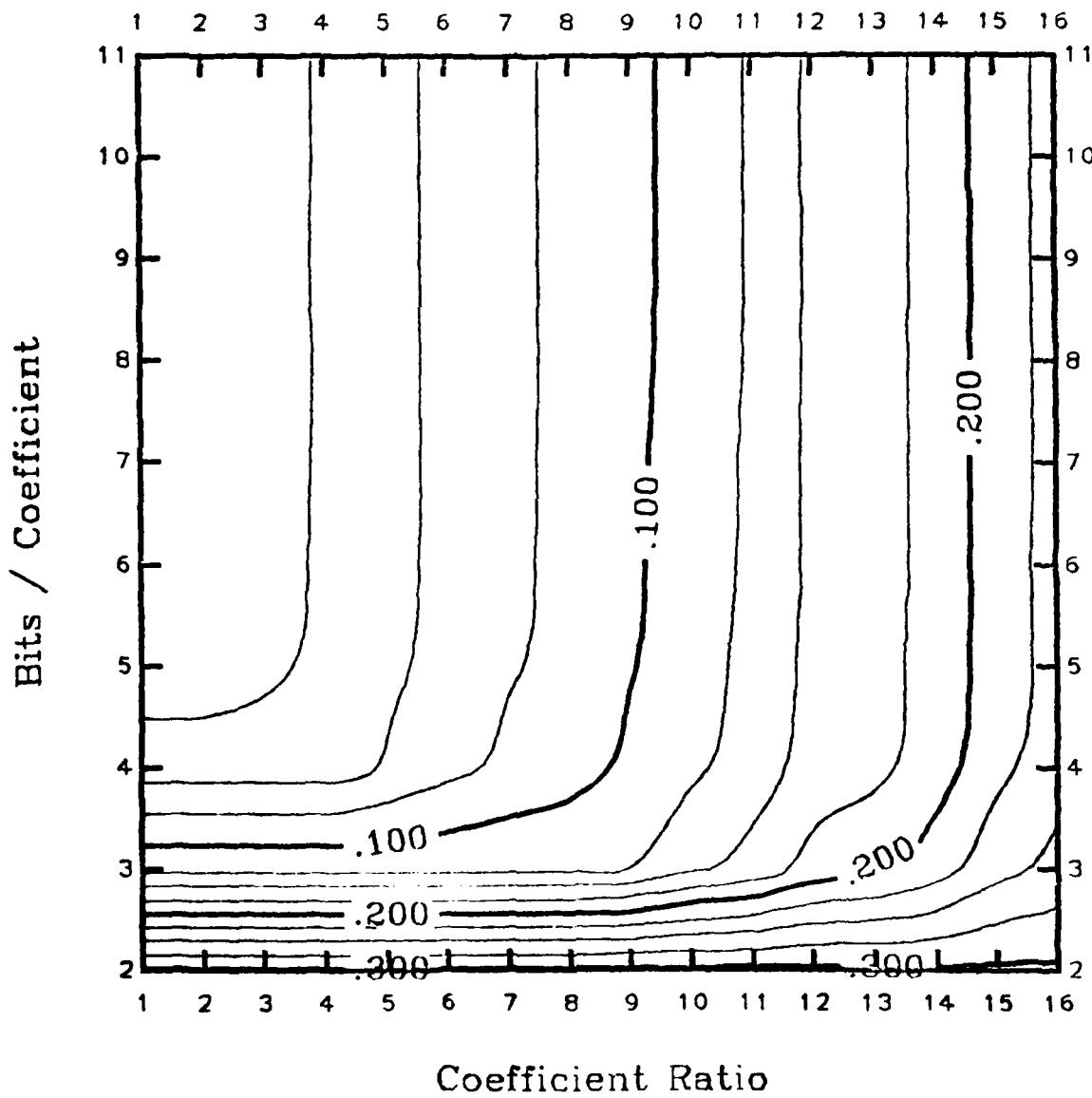


Figure 9. Contour plots of the normalized mean square error.

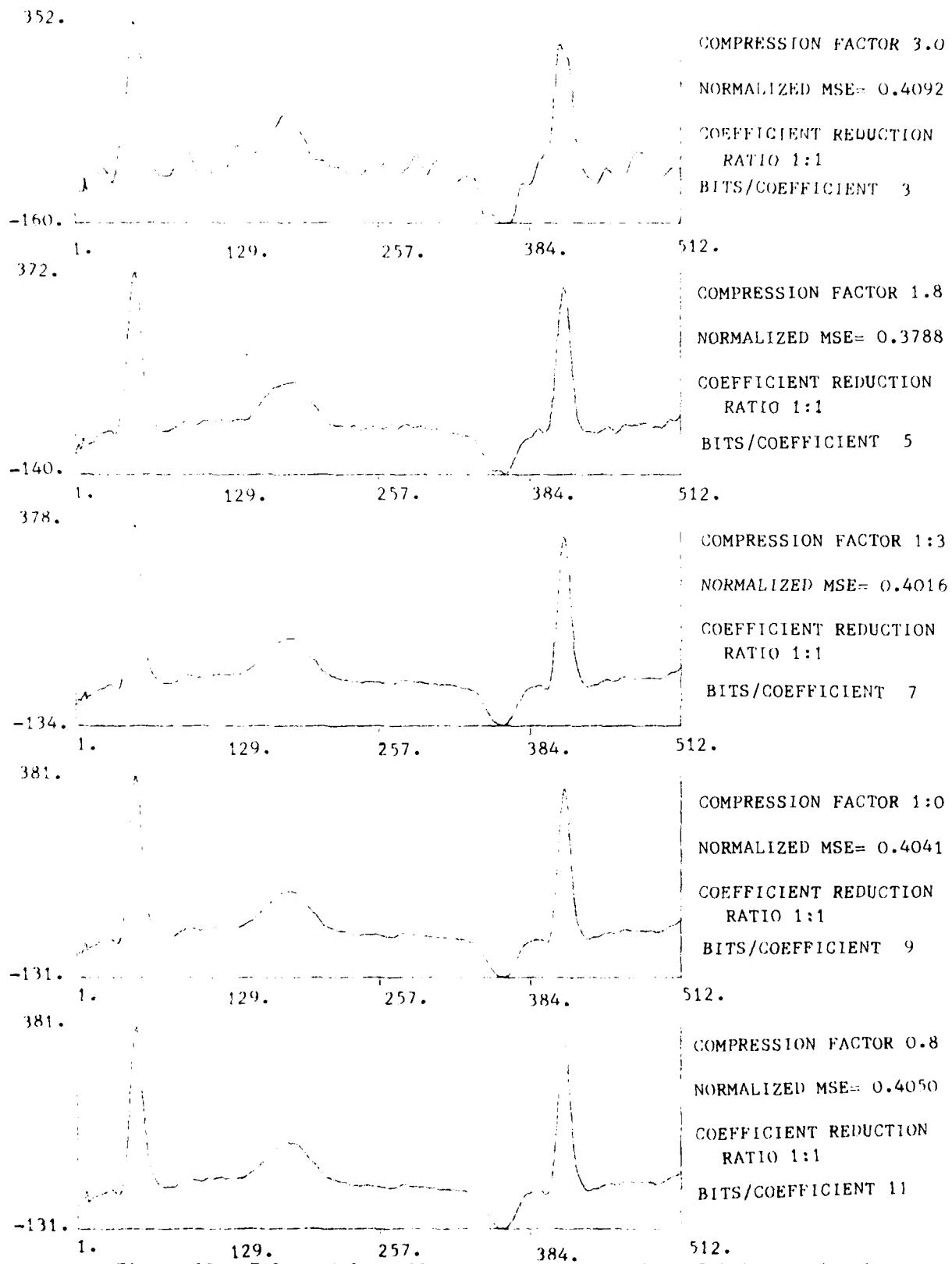


Figure 10. Filtered lead II reconstruction--1:1 coefficient reduction.

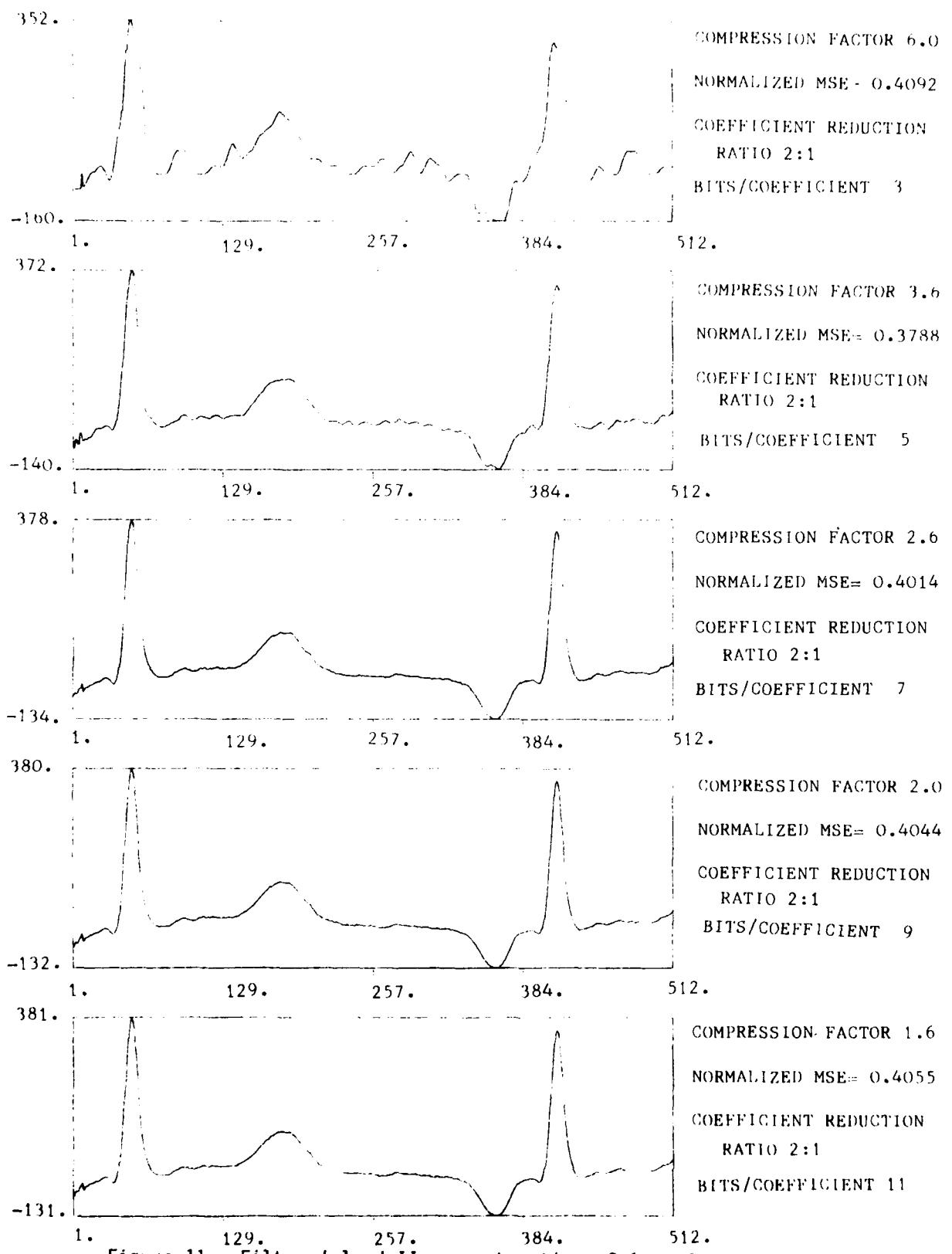


Figure 11. Filtered lead II reconstruction--2:1 coefficient reduction.

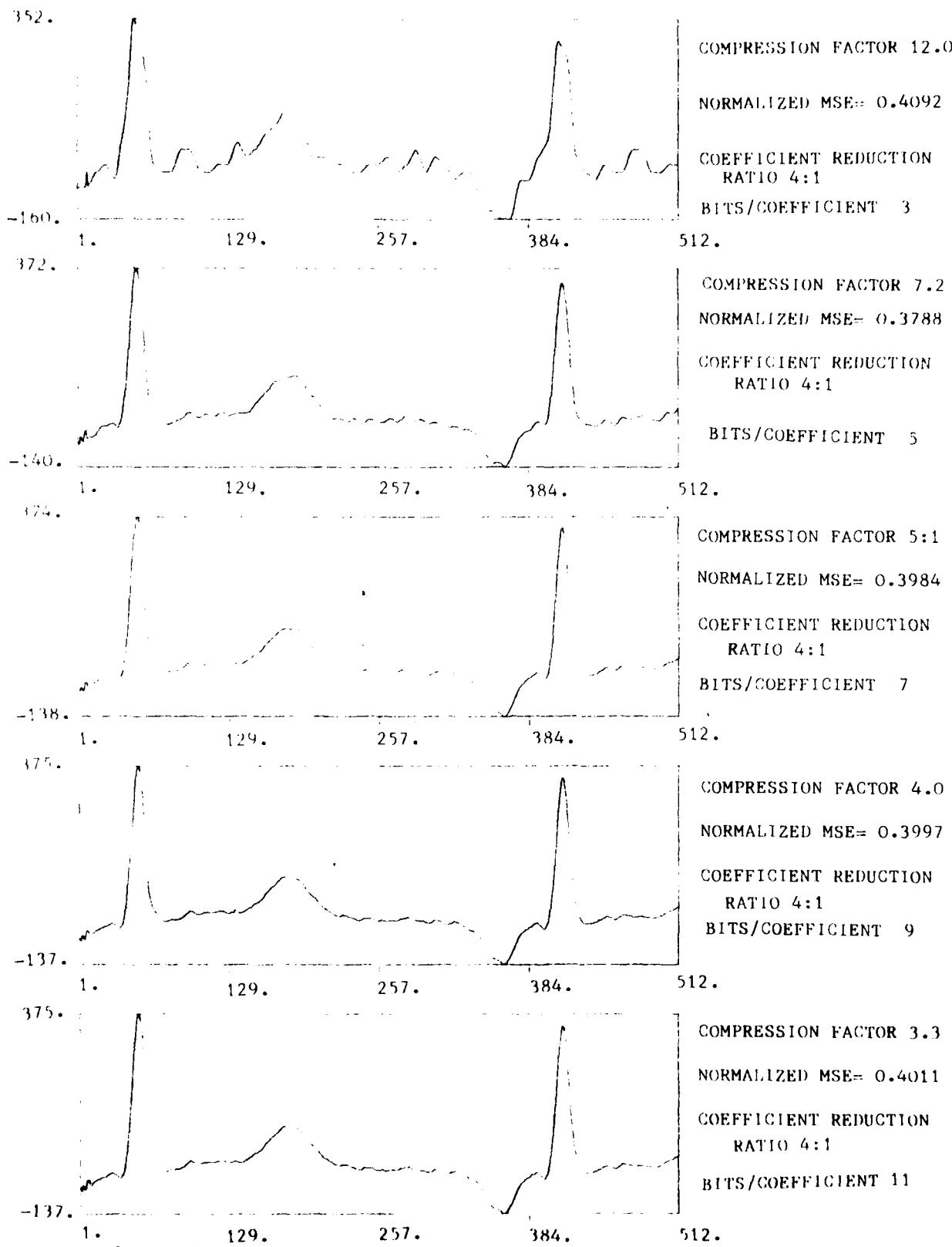


Figure 12. Filtered lead II reconstruction--4:1 coefficient reduction.

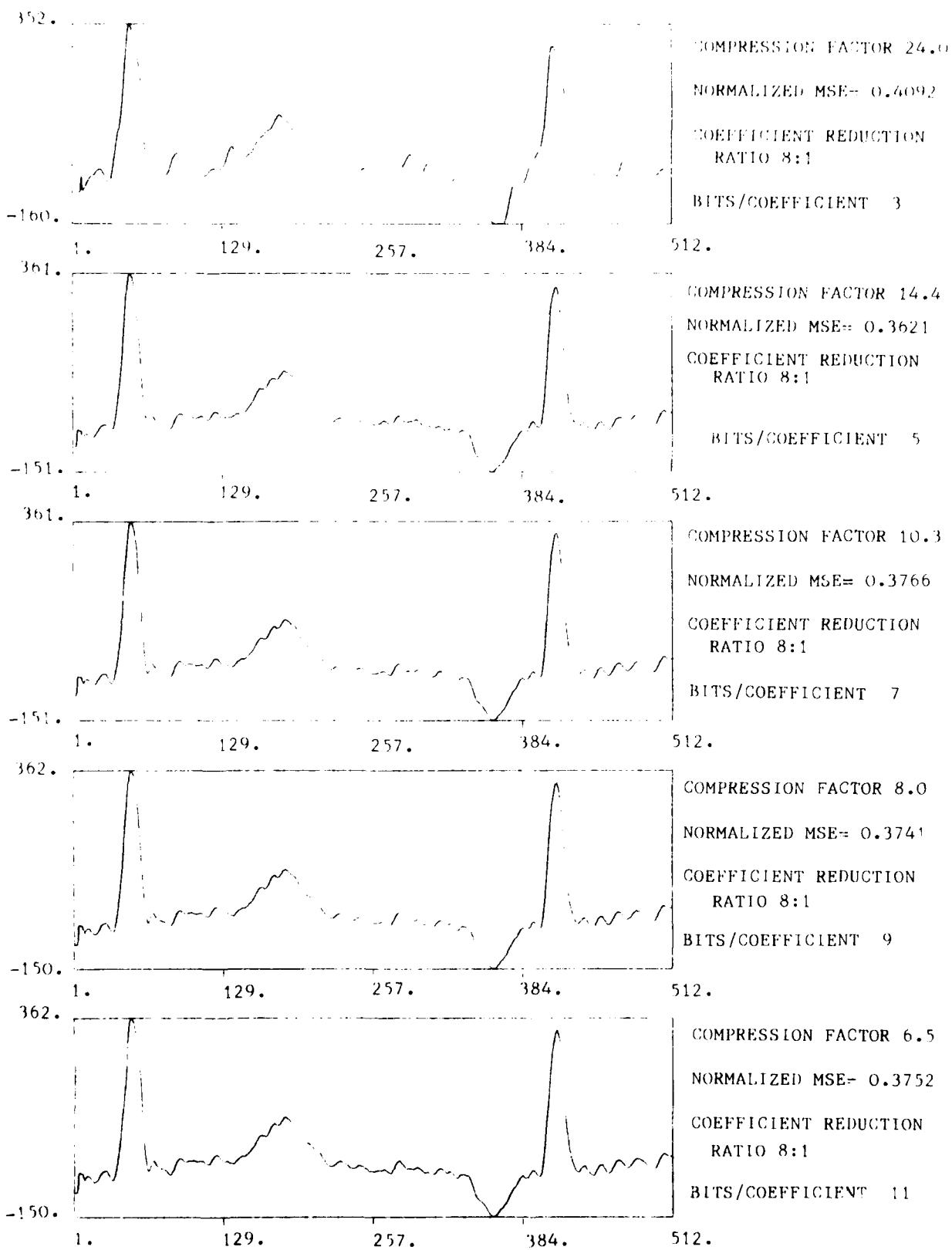


Figure 13. Filtered lead II reconstruction--8:1 coefficient reduction.

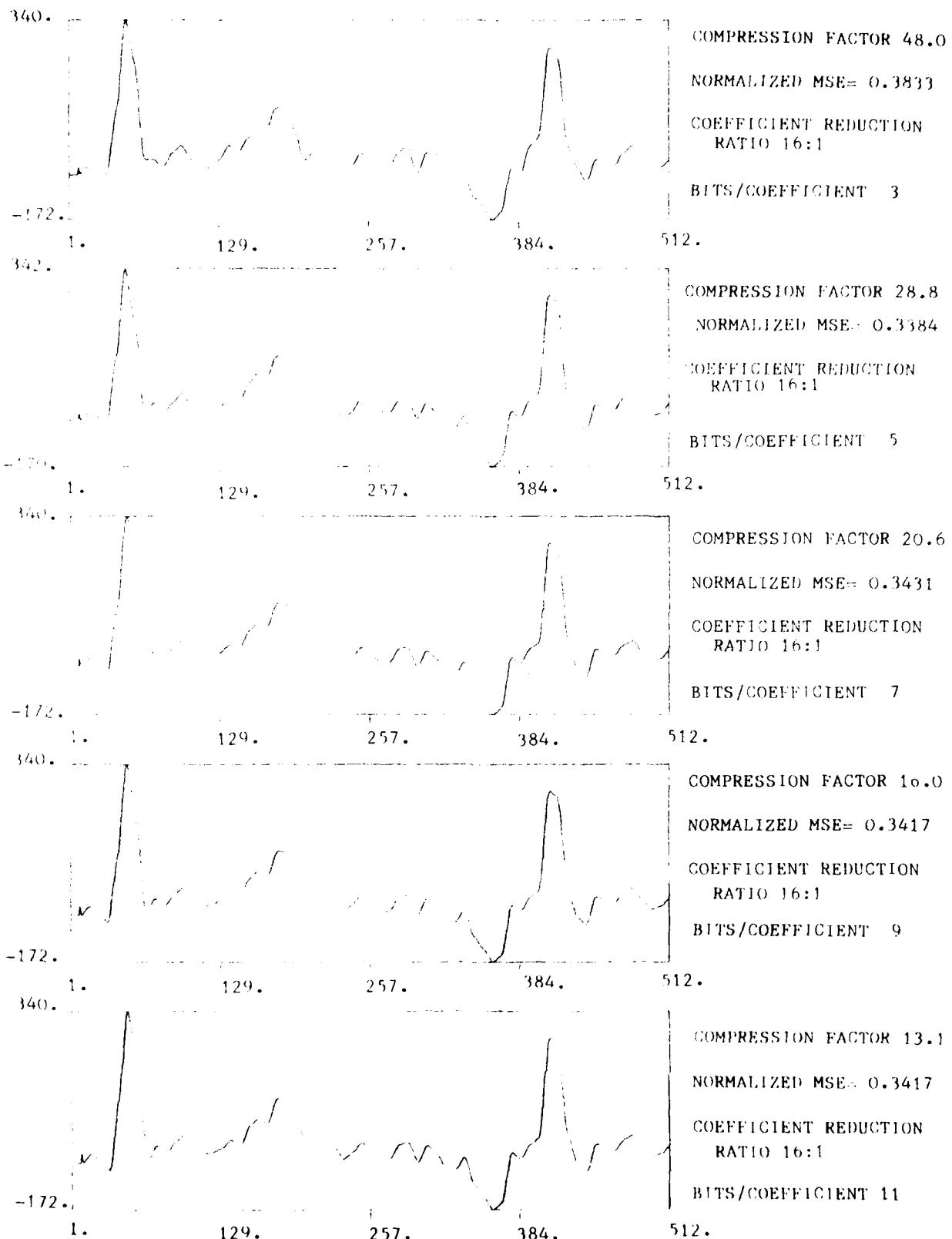
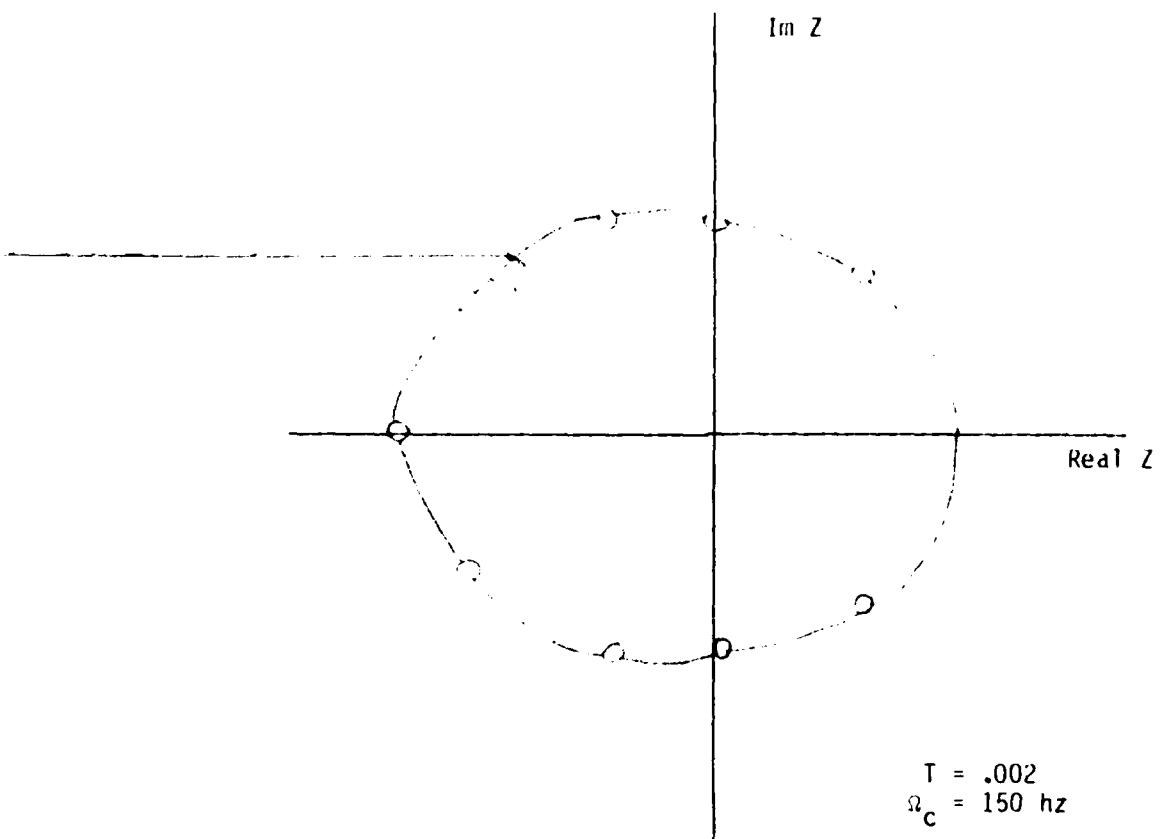


Figure 14. Filtered lead II reconstruction--16:1 coefficient reduction.



$$H(Z) = (Z+0.5+j0.866)(Z-0.5-j0.866)(Z+0.866+j0.5)(Z-j)(Z+1)$$

Figure 15. Filter transfer function--zero locations.

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